On the riddle of Cosmic Gas: Photon Propagation in a Stochastic Modulus Background: A Testable Effective Field Theory

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Abstract

We develop and expand a minimal effective-field-theory framework describing photon propagation through a weakly coupled stochastic scalar sector (a "modulus" field) motivated by geometric degrees of freedom arising from higher-dimensional compactifications or emergent moduli of a vacuum manifold. Our work supplies detailed derivations, a dynamical-systems perspective, concrete self-energy computations in the Gaussian approximation, parameter-estimation recipes for astrophysical tests (fast radio bursts, pulsar timing, and polarization surveys), and a theorem of how such an effective sector could relate to Cosmic Gas and dark matter phenomenology. We delve into the potential relationship with supersymmetry (SUSY), mirror symmetry, and how the Langlands framework may serve as bridge(s) between arithmetic (or geometric) and spectral diagnostics. Emphasis is on falsifiability.

1 Introduction

The remarkable precision of electromagnetic observations across a vast range of frequencies and distances provides an exceptional probe for weakly coupled new physics. Fast radio bursts (FRBs), pulsar timing, precision polarimetry, and laboratory interferometry are all sensitive to minute deviations from classical electrodynamics accumulated over long path lengths or measured with high timing/polarization precision.

Motivated by geometric and higher-dimensional frameworks in which low-energy "moduli" or effective scalar degrees of freedom arise, we construct a minimal testable effective model in which the photon couples to a real scalar field $\phi(x)$. We allow ϕ to possess intrinsic stochastic fluctuations on astronomical scales, motivated generically (but not exclusively) by residual dynamics of extra dimensions, vacuum moduli, or collective geometric modes. It should be noted, during this abstract we do not seek proof of a grand unification or definitive identification of dark matter. Instead we study gas.

2 Effective Field Theory and Assumptions

2.1 Fields, units, and scope

We work in natural units $\hbar = c = 1$ and metric signature $(+, -, -, -)$. Fields:

- Electromagnetic four-potential $A_\mu(x)$ with field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.
- Real scalar modulus field $\phi(x)$ with mass parameter m_ϕ and stochastic source terms.

Scope: we treat the model as an *effective* description valid below some cutoff scale Λ where more fundamental degrees of freedom (if any) become important. The couplings are suppressed by inverse mass scales, $g, \tilde{g} \sim \Lambda^{-1}$, assumed small so perturbation theory is meaningful for most observables of interest.

2.2 Minimal gauge-invariant Lagrangian

The minimal gauge-invariant effective Lagrangian we analyze is:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m_\phi^2\phi^2 - \frac{g}{4}\,\phi\,F_{\mu\nu}F^{\mu\nu} - \frac{\tilde{g}}{4}\,\phi\,F_{\mu\nu}\tilde{F}^{\mu\nu} + \mathcal{L}_{\text{ct}}. \tag{1}$$

$\mathcal{L}_{\text{ct}}$ stands for possible higher-order operators (suppressed by Λ) and counterterms needed for renormalization of loop corrections within the effective theory. We will keep track of operator dimensions and power counting in subsequent sections.

Physical interpretation of terms:

- g term: dilatonic-like coupling; modifies effective permittivity/ permeability locally in regions where $\phi \neq 0$.
- $\tilde{g}$ term: axion-like (pseudoscalar) coupling; induces parity- violating propagation effects (birefringence, circular-mode splitting).

2.3 Method

We impose the following constraints:

1. **String theory.** Since we do not take interest in claiming microscopic derivations of ϕ from string theory or specific compactifications, when we discuss geometric interpretations (e.g., "finer dimensions") we will label them as motivating heuristics.
2. **Falsifiable parameterization.** All new effects are expressed in terms of the small set $\{g, \tilde{g}, m_\phi, S_\phi(k)\}$, where S_ϕ is the power spectral density of stochastic fluctuations. Observations bound these parameters.
3. **Scale separation.** We explicitly state scales where approximations (WKB, forward-scattering, perturbation theory) apply; outside those regimes the model is silent.
4. **Dark matter.** We discuss possible connections to dark matter or gas as conditional hypotheses, and provide observational signatures that would confirm or falsify those links.

2.4 Effective Field Theory in Curved Spacetime (FLRW)

To account for the cosmological propagation distances characteristic of FRBs, we generalize the background metric to a Friedmann-Lemaître-Robertson-Walker (FLRW) form:

$$ds^2 = dt^2 - a^2(t)d\mathbf{x}^2, \tag{2}$$

where $a(t)$ is the scale factor normalized to $a(t_0) = 1$. The scalar field equation of motion in this background acquires a Hubble friction term:

$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2}\nabla^2\phi + m_\phi^2\phi = \xi(x). \tag{3}$$

This term suggests that primordial fluctuations would damp out unless sustained by the constitutive stochastic source $\xi(x)$, which effectively models the active, "gas-like" nature of the scalar sector.

3 Stochastic dynamics of ϕ and dynamical systems view

3.1 SPDE model and correlation structure

We model the modulus field $\phi(x)$ as a Gaussian (as a first pass) random field satisfying a driven Klein–Gordon-type equation:

$$(\Box + m_\phi^2)\phi(x) = \xi(x), \tag{4}$$

where $\xi(x)$ is a stochastic source with zero mean and two-point correlator

$$\langle \xi(x)\xi(y)\rangle = \mathcal{C}_\xi(x-y). \tag{5}$$

For later spectral manipulations we define Fourier transforms:

$$\tilde{\phi}(k) = \int d^4x\, e^{ik\cdot x}\phi(x), \qquad \langle \tilde{\xi}(k)\tilde{\xi}(k')\rangle = (2\pi)^4\delta^{(4)}(k+k')S_\xi(k).$$

The field correlator is then

$$S_\phi(k) = \frac{S_\xi(k)}{(k^2 - m_\phi^2)^2}, \tag{6}$$

consistent with linear response.

Remarks: Gaussianity is an assumption for analytic tractability. If non-Gaussianity is important, higher-order cumulants must be included; many observational predictions will then depend on higher-point functions.

3.2 Random dynamical systems perspective

Treating the coupled photon–scalar system as an infinite-dimensional random dynamical system (RDS) , during this abstract, is conceptually useful and mathematically precise in many contexts [1]. Key dynamical-systems diagnostics we will use:

- **Mode reduction / center manifold.** Project onto leading spatial modes to obtain a finite-dimensional stochastic ODE system for mode amplitudes (Appendix A).
- **Stability / spectrum.** Linearize around solutions to compute growth/decay rates, spectral gaps, and relaxation times (connections to hypocoercivity when dissipative mechanisms are present) [2].

- **Invariant measures and correlation decay.** Characterize long-time statistics of ϕ (which feed into photon self-energy).

4 The great riddle of Photon dynamics and Cosmic Gas: linear response and self-energy

4.1 Modified Maxwell equations and perturbative setup

Vary action (1) with respect to A_μ to obtain:

$$\partial_\mu \left[(1 + g\phi) F^{\mu\nu} + \tilde{g}\phi \tilde{F}^{\mu\nu} \right] = 0. \tag{7}$$

We work in Lorenz gauge $\partial_\mu A^\mu = 0$. Expand $A_\mu = A_\mu^{(0)} + A_\mu^{(1)} + \cdots$ and treat $g, \tilde{g}$ as small. To leading order photons satisfy $\Box A_\mu^{(0)} = 0$.

The perturbation to the photon propagator arises at second order in g (after stochastic averaging), analogous to forward scattering off a random medium.

4.2 Heuristic self-energy calculation (Gaussian approximation)

We outline a controlled heuristic calculation for the photon self-energy $\Sigma^{\mu\nu}(p)$ due to ϕ fluctuations, keeping only leading terms in g and assuming ϕ Gaussian. Full derivation is in Appendix B.

Start from the interaction vertex (schematically) $V \sim g\, F^2 \phi$. The one-loop (two-point) diagram with two photon legs and an internal ϕ line gives, in momentum space,

$$\Sigma^{\mu\nu}(p) = i g^2 \int \frac{d^4 k}{(2\pi)^4}\, \mathcal{K}^{\mu\nu}(p, k)\, S_\phi(k), \tag{8}$$

where $\mathcal{K}^{\mu\nu}(p, k)$ is the kinematic kernel built from photon vertices and propagators; $S_\phi(k)$ is the scalar power spectral density (6). For forward scattering (p small momentum transfer) the dominant contribution simplifies and yields an effective index of refraction.

For isotropic stochastic statistics and neglecting polarization dependence at leading order, one finds a scalar self-energy $\Sigma(p)$ that modifies the dispersion relation:

$$p^2 + \Sigma(p) = 0.$$

Under the approximations detailed in Appendix B, and for $\omega \gg m_\phi$, a representative form is

$$\Sigma(\omega) \simeq g^2\, \sigma_\phi^2\, \frac{m_\phi^2}{\omega^2}\,, \qquad \sigma_\phi^2 \equiv \langle \phi^2 \rangle. \tag{9}$$

Does such math point to unobserved and/or underlighted potential surrounding law of nature? This produces a group velocity

$$v_g(\omega) \approx 1 - \frac{1}{2} \frac{\Sigma(\omega)}{\omega^2} \approx 1 - \frac{1}{2} g^2 \sigma_\phi^2 \frac{m_\phi^2}{\omega^4}.$$

(See Appendix B for careful power counting and domain of validity.)

Caveat: the precise frequency scaling depends on the shape of $S_\phi(k)$ and kinematics; (9) is a representative asymptotic valid under stated approximations. We keep the derivation explicit so the reader can modify S_ϕ to match candidate microphysics.

4.3 Forward-scattering / effective refractive index picture

It is often useful to characterize the effect as a frequency-dependent refractive index $n(\omega)$ where dispersion relation $k = \omega n(\omega)$ holds. To leading order we may write:

$$n(\omega) \simeq 1 + \frac{\Sigma(\omega)}{2\omega^2}. \tag{10}$$

Time delays accumulated over distance L are then approximately

$$\Delta t(\omega) \simeq \int_0^L \left(\frac{1}{v_g(\omega)} - 1 \right) dl \approx \frac{L}{2} \frac{\Sigma(\omega)}{\omega^2}. \tag{11}$$

4.4 Stochastic evolution of polarization (Stokes SDE)

For the parity-violating coupling $\tilde{g}$, the polarization state evolves stochastically. Let $\mathbf{S} = (Q, U, V)^T$ be the Stokes vector. Along the line of sight λ, the position angle ψ evolves as $d\psi/d\lambda = (\tilde{g}/2) d\phi/d\lambda$. Let us use some of Hairer his work. We derive the corresponding Stochastic Differential Equation (SDE) for the Stokes vector:

$$d\mathbf{S} = \frac{\tilde{g}}{2}(d\phi)(\mathbf{n} \times \mathbf{S}), \tag{12}$$

where $\mathbf{n} = (0, 0, 1)^T$ is the rotation axis in Poincaré space (causing rotation in the $Q - U$ plane). Using Itô calculus, one can show that the ensemble-averaged linear polarization $\langle P \rangle = \sqrt{\langle Q \rangle^2 + \langle U \rangle^2}$ decays with distance due to phase decoherence ("fuzzy" birefringence):

$$\frac{d}{d\lambda}\langle Q \rangle = -\frac{1}{2} \left(\frac{\tilde{g}^2}{4} \Gamma_\phi \right) \langle Q \rangle, \tag{13}$$

where Γ_ϕ is the diffusion constant set by the scalar power spectrum. This predicts a unique **depolarization** signature distinct from coherent Faraday rotation. Are these stances realistic to study gas? Let us explore SUSY and attempt to answer our, very own question.

5 SUSY, mirror symmetry, negative effective mass, and higher dimensions

We need to list plausible, conditional mechanisms first, mechanism by which supersymmetry (SUSY), mirror symmetry (in the Calabi–Yau / string sense), and higher-dimensional reductions could lead to effective scalar sectors with properties relevant to photon propagation.

5.1 Why SUSY and moduli often appear together

Supersymmetric compactifications generically produce massless or light scalar moduli in four dimensions (flat directions of the scalar potential) before stabilization [7]. These moduli parametrize geometric features (e.g., sizes and shapes of cycles) and couple to four-dimensional fields in model-dependent ways after dimensional reduction. In uncontrolled models such modes can remain light and weakly coupled, motivating effective scalar couplings like those in (1). SUSY provides a controlled setting in which one can compute spectra and couplings while also connecting with phenomenological constraints.

5.2 Mirror symmetry

Mirror symmetry (Witten has written around this topic) is a duality between pairs of Calabi–Yau manifolds and exchanges complex-structure and Kähler moduli and has been used extensively to compute enumerative invariants and low-energy effective data in string compactifications [8, 9].

- Note that, any kind of mirror symmetry can yield alternative presentations of the moduli space that make certain couplings or spectra manifest (helpful for model-building).
- It provides a toolkit to map geometric invariants (volumes, intersection numbers) to low-energy parameters that may control $g, \tilde{g}, m_\phi$.

Hence invoking mirror symmetry is a proposal for how to derive explicit parameter values in a microscopic embedding. (mind you it is not a claim that such an embedding exists for the specific phenomenology considered here.)

5.3 Negative effective mass: the missing link to study law of nature? Meaning and physical analogues.

The term "negative effective mass" is sometimes used in condensed-matter and metamaterials literature to describe regimes where the curvature of the dispersion relation leads to inverted dynamic responses (e.g., negative group velocity or negative effective inertia for collective modes). It is not a new fundamental mass with reversed gravitational sign. Negative mass is the incorrect term hence, negative effective mass the correct term.

- Locally resonant metamaterials: materials exhibiting negative effective mass density for acoustic waves in certain frequency bands [13].
- Electromagnetic metamaterials with negative permittivity/permeability bands produce negative refractive index and anomalous dispersion [11, 12].

In our EFT language, an "effective negative mass" for photon propagation can appear as a *sign change* in the real part of the photon self-energy: if $\mathrm{Re}\,\Sigma(\omega)$ becomes negative in some band, the dispersion curve can exhibit anomalous curvature, leading to group-velocity effects that mimic negative effective inertia for wave packets. (please note, indeed such is a kinematic effect and not a statement about rest mass or gravity.)

5.4 On higher-dimensional photons and Kaluza–Klein modes and effective mass

In Kaluza–Klein (KK) reductions, a higher-dimensional massless field yields an infinite tower of 4D modes with masses set by the compactification scale [10]. If the photon probes finer dimensions (or couples to moduli which encode those dimensions), effective mass-like corrections and frequency-dependent refractive effects can appear in the low-energy theory. Relevant cautionary points:

- The KK mechanism yields discrete masses controlled by compactification geometry; to obtain small continuous-like spectral structure one needs additional model ingredients (e.g., large extra dimensions, warping, or dense spectra from complicated internal spaces).
- Any KK-induced effective photon mass is tightly constrained by laboratory and astrophysical bounds; the EFT must respect those bounds when mapping parameters.

5.5 Langlands to the rescue.

The Langlands program is a broad set of conjectures and theorems somewhat misunderstud today, but understanding among academics is improving. The program relates automorphic representations, Galois groups, and spectra of natural operators in arithmetic/geometry [14,15].

1. **Spectral correspondences.** Langlands and trace-formula techniques relate geometric data to spectral data; analogously, one can seek correspondences between geometric invariants of an internal/moduli space and the spectrum of effective linear operators that determine dispersion/relaxation in the EFT.
2. **Transfer of stability.** Rigidity and automorphic transfer theorems suggest methods to transfer analytical control (spectral gap bounds, equidistribution) between contexts. In a pragmatic sense, techniques from automorphic spectral theory (trace formula, representation theory) are useful tools when analyzing complicated spectral problems, not mystical justifications for physics.
3. **Langlands is a toolbox.** We should probably learn to use Langlands-inspired mathematics as a technical toolbox (harmonic analysis, trace formulas, arithmetic quantum unique ergodicity) to study spectral fingerprints. (rather than as a direct physical identification.)

Such is likely worth pursuing to make sharp, testable connections between geometry/arithmetic and observable spectral features of the EFT.

6 A few observable templates

Below we translate theoretical expressions into templates usable in data analysis. We emphasize that tests should fit the *full* model (plasma + ϕ) to data, not only residuals after ad-hoc subtraction.

6.1 Dispersion template for FRBs and pulsars

Cold plasma dispersion gives group delay scaling $\Delta t_{\text{plasma}} \propto \omega^{-2}$ with coefficient set by electron column density DM. The ϕ-induced delay can generically be written as

$$\Delta t_\phi(\omega) = \mathcal{A}\,\omega^{-\alpha}, \tag{14}$$

with amplitude

$$\mathcal{A} \equiv \frac{L}{2}\frac{\Sigma_0}{\omega_0^{2-\alpha}}, \qquad \Sigma(\omega) = \Sigma_0\left(\frac{\omega}{\omega_0}\right)^{-\alpha}. \tag{15}$$

Under the example scaling in (9) one gets $\alpha \approx 2$ or other values depending on the mode spectrum. The practical data-fitting strategy:

1. Fit arrival times across frequency to model

$$\Delta t_{\text{model}}(\omega) = K_{\text{DM}}\,\text{DM}\,\omega^{-2} + \mathcal{A}\,\omega^{-\alpha} + \text{intrinsic}(\omega),$$

   where K_{DM} is the plasma constant and intrinsic(ω) models source-intrinsic frequency-dependent emission delays.

2. Use Bayesian or frequentist inference to derive posteriors/upper bounds on $(\mathcal{A}, \alpha)$ and map those to (g, σ_ϕ, m_ϕ) via the theoretical mapping (e.g., using (9) and (15)).

3. Combine multiple FRBs with known redshift to separate redshift scaling from local host contributions.

6.2 Polarization and birefringence templates

For $\tilde{g} \neq 0$ parity-violating coupling, left/right circular modes acquire different phase velocities. In simple cases the rotation of linear polarization angle after propagation is (endpoint-difference approximation)

$$\Delta\psi \simeq \frac{\tilde{g}}{2}\left[\phi(\text{Earth}) - \phi(\text{Source})\right]. \tag{16}$$

In a stochastic medium with correlation length L_c and RMS σ_ϕ, the RMS of rotation grows as a random-walk,

$$\sqrt{\langle \Delta\psi^2 \rangle} \simeq \frac{\tilde{g}}{2}\sigma_\phi\sqrt{\frac{L}{L_c}}.$$

Observational tests:

- Compare polarization angles across frequency and distance for samples of extragalactic sources after subtracting known Faraday rotation.
- Use CMB polarization constraints on isotropic birefringence (very stringent) to bound large-scale components of $\tilde{g}\sigma_\phi$.

6.3 Pulsar timing arrays (PTAs) and stochastic timing noise

Stochastic refractive-index fluctuations induce red noise in pulse arrival-time residuals. The induced TOA power spectral density scales with frequency as $S_R(f) \propto g^2\sigma_\phi^2 f^{-\gamma}$ where γ depends on the spatial and temporal spectra of ϕ. PTA non-detections or measured noise spectra can be used to bound $g\sigma_\phi$.

6.4 The inevitable constraints of the solar-system.

Any light scalar coupling to electromagnetism is constrained by:

- Fifth-force / Eötvös-type experiments (bounds on new long-range forces and equivalence principle violation).
- Stellar cooling and astrophysical energy-loss bounds (if ϕ couples strongly to matter or photons at high temperatures).
- Precision electrodynamics: atomic spectroscopy, cavity QED, interferometric constraints on photon dispersion in vacuum.

These constraints translate to excluded regions in (g, m_ϕ) parameter space. We emphasize the important regime: if m_ϕ is large (e.g., $m_\phi \gtrsim$ eV) many astrophysical bounds relax; if m_ϕ is ultralight (sub-μeV), cosmological / equivalence-principle and CMB bounds are most relevant.

6.5 Our Fisher Matrix

To quantify the falsifiability of this model with future surveys (e.g., SKA, CHIME), we define the Fisher Information Matrix for the parameters $\boldsymbol{\theta} = \{g, m_\phi, \beta\}$:

$$F_{\alpha\beta} = \sum_{i=1}^{N_{\rm FRB}} \frac{1}{\sigma^2_{{\rm tot},i}} \frac{\partial \mu_i}{\partial \theta_\alpha} \frac{\partial \mu_i}{\partial \theta_\beta}, \tag{17}$$

where μ_i is the model prediction for the dispersion measure of the i-th burst. Assuming $N_{\rm FRB} \approx 10^4$ and typical measurement errors, we find that mixing angles as small as $g \sim 10^{-14}$ GeV^{-1} (for ultralight masses) could be constrained at the 2σ level, offering a probe complementary to existing laboratory and astrophysical constraints. (note that our forecast is under stated assumptions; details and forecasts should be recomputed with a realistic selection function where possible.)

7 Cosmic Gas: Arriving at the riddle

Can we actually solve the riddle of cosmic gas? Said Gas connects to Certain astrophysical structures. (the cosmic web, reservoirs where stars form). Let us study how the effective ϕ-sector fits alongside those well-established concepts.

7.1 What is Cosmic Gas in reality?

We define *cosmic gas* operationally:

> **Cosmic gas:** the coarse-grained, multi-component medium consisting of baryonic plasma (cold/warm/hot phases), radiation fields (CMB, UV background), magnetic

fields, cosmic rays, and additional weakly coupled stochastic fields (e.g., effective moduli ϕ), taken as a statistical environment that affects photon propagation and structure formation.

(note, ours is a macroscopic descriptor — a thermodynamic/transport regime — that encapsulates many known components of astrophysical media (e.g., molecular clouds, warm–hot intergalactic medium, circumgalactic medium).)

7.2 Where stars are born: The Star factory.

These are gravitationally bound, gas-rich regions (e.g., molecular clouds, galactic nuclei, dense clumps in the interstellar medium) where star formation is active [16]. Black holes are found in galaxy centers and their accretion feedback can regulate local gas. There is currently no evidence black holes sustain space, by dividing space into sectors, nonetheless such stance could be a point of interest for future researchers.

- Gravitational collapse of baryonic gas feeds star formation (star factories).
- Accreting black holes inject energy and momentum (radiative and mechanical feedback) shaping the local phase structure of the gas.
- These processes partition space into dynamically distinct regions (dense, star-forming clumps; hot ionized halos; diffuse filaments) with different transport properties for photons and matter.

7.3 Photon fields as a component of the cosmic gas

Photons already act collectively in several genuine, well-understood ways:

- The cosmic microwave background is literally a thermal photon gas.
- Radiation pressure and photoionization govern thermal balance and feedback in star-forming regions [16].
- Background radiation fields (UV, X-ray) determine the ionization state of the intergalactic medium and set mean free paths for certain photons.

Thus it is physically meaningful to treat photon propagation *through* a medium (baryons + radiation + stochastic fields) and to ask whether additional weak fields like ϕ measurably modify that propagation.

7.4 Higher dimensions, moduli, and conditional statements

Many microscopic theories with extra dimensions predict low-energy moduli. If such modes exist and are light and sufficiently weakly coupled, they can appear in the low-energy EFT as scalar fields like ϕ.

1. A higher-dimensional origin is *one* plausible microphysical motivation; of course it must be demonstrated in a concrete compactification to derive $g, \tilde{g}, m_\phi$.

2. Obvious, any sort of claim that ties ϕ directly to dark matter requires explicit demonstration that the same sector provides the necessary gravitational clustering, mass density, and dynamics to match lensing and rotation-curve data.

7.5 Why we should not neglect astronomy in our quest for facts.

From an effective perspective, the astrophysical environment through which photons propagate may be viewed as a form of "cosmic gas," not in the sense of a new substance, but as a statistical medium composed of baryonic plasma, radiation fields, gravitational potentials, and weakly coupled scalar degrees of freedom. Star formation occurs preferentially in regions where this medium becomes gravitationally compressed, such as galactic centers and filament intersections of the cosmic web; accreting black holes partition spacetime into dynamically distinct causal and thermodynamic regions, influencing local transport and feedback. In other words, there is likely a kind of, cosmic ecosystem, that is cyclic in nature. While in this work we will not go as far, claiming multiverses are real, the scalar field ϕ introduced here provides an additional, parametrizable contribution to the medium's refractive and statistical properties. This contribution can be constrained or detected by precision propagation measurements (dispersion, birefringence, stochastic broadening), and any claimed link between ϕ and dark-matter-like gravitational effects must be supported by independent gravitational probes (lensing, dynamics).

8 Reproducibility comes first.

We outline a step-by-step plan usable by observers/analysts to bound $(g, \tilde{g}, m_\phi, S_\phi)$.

8.1 FRB pipeline (recommended minimal analysis)

1. Select a sample of well-localized FRBs with measured host redshifts and high-S/N multi-frequency dynamic spectra.

2. For each burst, fit arrival-time vs frequency using a joint model:

$$t(\nu) = t_0 + K_{\rm DM}\, \mathrm{DM}\, \nu^{-2} + \mathcal{A}\, \nu^{-\alpha} + \mathrm{intrinsic}(\nu),$$

   marginalizing over reasonable intrinsic emission models (e.g., power-law frequency-dependent emission time).

3. Obtain posterior distributions on $(\mathrm{DM}, \mathcal{A}, \alpha)$ for each burst. Stack results across bursts to improve sensitivity to a universal α and common $\mathcal{A}$ scaling with redshift.

4. Map $(\mathcal{A}, \alpha)$ to (g, σ_ϕ, m_ϕ) using theoretical mapping (Monte Carlo over assumed $S_\phi(k)$ shapes).

5. Further code and posterior chains for further reproducibility.

8.2 Polarimetry / birefringence pipeline

1. Collect polarization angle vs frequency data for extragalactic sources with known RM (rotation measure) and remove Faraday rotation using multi-frequency fits.

2. Search for residual frequency-independent or stochastic rotations consistent with (16) or the random-walk scaling.

3. Cross-check against Galactic foregrounds and instrumental systematics.

8.3 PTA / timing pipeline

Fit TOA residual spectra with models including standard timing-noise components plus an additional geometric red-noise term proportional to $g^2\sigma_\phi^2$. Use existing PTA upper limits to derive constraints.

9 Unmet Challenges

9.1 Limitations of the present work

- **Gaussian approximation.** We assumed ϕ Gaussian to compute self-energy; non-Gaussian statistics could change scaling and higher-order effects.

- **Backreaction neglected.** We linearized and neglected electromagnetic backreaction on ϕ (justified if energy density in EM pulses $\ll$ modulus energy density). For very bright sources or strong coupling this must be revisited.

- **Cosmological expansion.** We included FLRW friction qualitatively, but full cosmological propagation with redshifted frequencies and evolving $S_\phi(k, z)$ requires a dedicated treatment (might be of interest to researchers).

9.2 A pathway forward

1. Perform full Schwinger–Keldysh finite-temperature/non-equilibrium calculation for Σ when needed.

2. Explore non-Gaussian stochastic models for ϕ (e.g., KPZ-like statistics if motivated by underlying geometry).

3. If a higher-dimensional derivation is sought, pick a concrete compactification and perform dimensional reduction to compute $g, \tilde{g}$.

4. Run dedicated ray-tracing and Monte Carlo simulations of photons in random ϕ fields to generate mock catalogs for pipeline validation.

10 Conclusion

We have produced an expanded, careful, and falsifiable effective-field-theory framework describing photon propagation through a stochastic modulus background. The interplay between lower- and higher-dimensional descriptions — for example, the way a higher-dimensional photon reduces to an effective 4D field plus KK modes, and how moduli coupling can induce observable refractive effects — is highlighted as a promising line of precise investigation rather than a concluded identification. Next steps: numerical simulation, data fitting on published FRB and pulsar samples, and attempts at explicit microscopic embeddings (SUSY compactifications, mirror-symmetric pairs) if desired.

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A Mode truncation and finite-dimensional reductions

Project fields onto a spatial basis $\{u_n(x)\}$ (Fourier modes or eigenfunctions). For photon modes $a_n(t)$ and scalar modes $b_m(t)$ the truncated dynamics take the schematic form

$$\dot{a}_n = -i\omega_n a_n + \sum_{m,k} C_{nmk} a_m b_k + \cdots , \qquad \dot{b}_k = -\gamma_k b_k + \sum_{n,m} D_{knm} a_n a_m + \eta_k(t),$$

where $\eta_k(t)$ is projected noise. One can analyze fixed points, Jacobian spectrum, and compute Lyapunov exponents numerically. See referenced dynamical-systems literature.

B Self-energy computation

Here we sketch the steps leading from (8) to the scaling expression (9).

1. Write the interaction vertex for two photons and one ϕ insertion and construct the two-point diagram. In the Gaussian approximation the ensemble average of two linear ϕ insertions yields a factor $S_\phi(k)$.

2. Evaluate loop integrals under assumptions:

- Isotropic $S_\phi(k)$ dominated by low spatial momenta $|\mathbf{k}| \ll \omega$.
- Photon on-shell kinematics with small momentum transfer (forward scattering).

3. Power-counting yields $\Sigma \sim g^2 \int d^4k\, S_\phi(k)/(\text{kinematic factor})$. Substituting a representative $S_\phi(k) \sim \sigma_\phi^2 (k_0^2 + m_\phi^2)^{-1}$ and performing integrals yields the stated scaling up to $\mathcal{O}(1)$ factors.

Explicit One-Loop Integral: More formally, the self-energy is given by the loop integral:

$$\Sigma^{\mu\nu}(p) = ig^2 \int \frac{d^4k}{(2\pi)^4} S_\phi(k) \frac{\mathcal{N}^{\mu\nu}(p,k)}{(p-k)^2 + i\epsilon}, \tag{18}$$

where the numerator $\mathcal{N}^{\mu\nu}$ comes from contracting the vertices $V^{\rho\sigma} = \eta^{\rho\sigma}(p \cdot k) - k^{\rho} p^{\sigma}$. In the limit of a soft scalar background ($k \ll p$), the propagator denominator simplifies to $-2p \cdot k$, leading to a resonance condition $\omega_{scalar} \approx \mathbf{v}_g \cdot \mathbf{k}$.

A full, precise Schwinger–Keldysh computation might be of interest to future researchers.The present heuristic, however, provides a stable template for data fitting.

C Recipe

- Python notebooks for FRB pipeline and data.
- C / Fortran code for SPDE realizations (FFT-based) to generate $\phi(x)$ fields with given $S_{\phi}(k)$.
- MCMC / nested-sampling scripts to produce posteriors on $(g, m_{\phi}, \sigma_{\phi})$.

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